Corrigé : Evaluation n° 1

Exercice 1

Calculer les dérivées partielles à l'ordre 1 et 2 des fonctions suivantes :

1. $f(x,y) = 4x^3 - 2x^2 - 5xy + 8x^2y$

$$\frac{\partial f}{\partial x}(x,y) = 12x^2 - 4x - 5y + 16xy \quad \text{et} \quad \frac{\partial f}{\partial y}(x,y) = -5x + 8x^2$$
$$\frac{\partial^2 f}{\partial x^2}(x,y) = 24x - 4 + 16y \quad \text{et} \quad \frac{\partial^2 f}{\partial y^2}(x,y) = 0$$
$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = -5 + 16x$$

2. $f(x,y) = -x^4 + x^3y + \frac{1}{6}y^3 + 8$

$$\frac{\partial f}{\partial x}(x,y) = -4x^3 + 3x^2y \quad \text{et} \quad \frac{\partial f}{\partial y}(x,y) = x^3 + \frac{1}{2}y^2$$
$$\frac{\partial^2 f}{\partial x^2}(x,y) = -12x^2 + 6xy \quad \text{et} \quad \frac{\partial^2 f}{\partial y^2}(x,y) = y$$
$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = 3x^2$$

3. $f(x,y) = x^2\ln(y) + \mathrm{e}^x y^3$

$$\frac{\partial f}{\partial x}(x,y) = 2x\ln(y) + \mathrm{e}^x y^3 \quad \text{et} \quad \frac{\partial f}{\partial y}(x,y) = \frac{x^2}{y} + 3\mathrm{e}^x y^2$$
$$\frac{\partial^2 f}{\partial x^2}(x,y) = 2\ln(y) + \mathrm{e}^x y^3 \quad \text{et} \quad \frac{\partial^2 f}{\partial y^2}(x,y) = \frac{-x^2}{y^2} + 6\mathrm{e}^x y$$
$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{2x}{y} + 3\mathrm{e}^x y^2$$

4. $f(x,y) = \dfrac{x^4}{y}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{4x^3}{y} \quad \text{et} \quad \frac{\partial f}{\partial y}(x,y) = \frac{-x^4}{y^2}$$
$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{12x^2}{y} \quad \text{et} \quad \frac{\partial^2 f}{\partial y^2}(x,y) = \frac{2x^4}{y^3}$$
$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{-4x^3}{y^2}$$

5. $f(x,y) = \ln(x+y)$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{x+y} \quad \text{et} \quad \frac{\partial f}{\partial y}(x,y) = \frac{1}{x+y}$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{-1}{(x+y)^2} \quad \text{et} \quad \frac{\partial^2 f}{\partial y^2}(x,y) = \frac{-1}{(x+y)^2}$$
$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{-1}{(x+y)^2}$$

6. $f(x,y) = x\sqrt{y} + y^2\sqrt{x}$

$$\frac{\partial f}{\partial x}(x,y) = \sqrt{y} + \frac{y^2}{2\sqrt{x}} \quad \text{et} \quad \frac{\partial f}{\partial y}(x,y) = \frac{x}{2\sqrt{y}} + 2y\sqrt{x}$$
$$\frac{\partial^2 f}{\partial x^2}(x,y) = y^2 \times \frac{\frac{-2}{2\sqrt{x}}}{(2\sqrt{x})^2} = \frac{-y^2}{4x\sqrt{x}} \quad \text{et} \quad \frac{\partial^2 f}{\partial y^2}(x,y) = x \times \frac{\frac{-2}{2\sqrt{y}}}{(2\sqrt{y})^2} + 2\sqrt{x} = \frac{-x}{4y\sqrt{y}} + 2\sqrt{x}$$
$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{1}{2\sqrt{y}} + \frac{2y}{2\sqrt{x}} = \frac{1}{2\sqrt{y}} + \frac{y}{\sqrt{x}}$$